Music Theory

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Chapter 1

Sound and Musical Scales

1.1 Sound

From the study of modern science, it has been found that sound is a physical phenomenon involving pressure waves in a medium, typically the molecular nitrogen and molecular oxygen atmosphere of planet Earth[1]. In this book, we shall be interested only in the frequencies of these pressure waves.

1.2 Scales

1.2.1 Chromatic Scale

Definition 1. *octave;*
Given two frequencies, f_1 and f_0, if

$$f_1 \quad = \quad 2f_0$$

, then f_1 is said to be an octave above f_0, and, f_0 is said to be an octave below f_1.

Definition 2. *chromatic;*
Given a frequency f and another frequency f_0, called the root frequency, if f may be given by

$$f \quad = \quad f_0 \cdot 2^{\frac{n}{12}}$$

where $n \in \mathcal{I}$, then f is said to be a chromatic frequency and n is said to be the chromatic number.

[1]The reader is referred to introductory physics text books that cover this topic. Such books typically require a level of reader sophistication ranging from senior high school to that of first year college or university.

Using Definitions 1 and 2, let us now use the following table to give names of the chromatic frequencies within in one octave, i.e. frequencies with chromatic number $n : 0 \leq n \leq 12$.

n	name	symbol
0	root	(1)
1	minor 2	(♭2)
2	major 2	(2)
3	minor 3	(♭3)
4	major 3	(3)
5	perfect 4	(4)
6	diminished 5 or augmented 4	(♭5) or (♯4)
7	perfect 5	(5)
8	minor 6	(♭6)
9	major 6	(6)
10	minor 7	(♭7)
11	major 7	(7)
12	root of next octave	(1)

1.2.2 Diatonic Scales

Using the chromatic scale defined in Section 1.2.1, let us now define the 7 diatonic scales.

Ionian Scale

The ionian scale contains (1)(2)(3)(4)(5)(6)(7)(1), where the first (1) indicates root and the final (1) indicates root of the next octave.

Dorian Scale

The dorian scale may be obtained by making the (2) of the ionian mode the root frequency, and maintaining the same intervals between the chromatic numbers. Therefore, the dorian scale contains (1)(2)(♭3)(4)(5)(6)(♭7)(1), where the first (1) indicates root and the final (1) indicates root of the next octave.

Phrygian Scale

The phrygian scale may be obtained by making the (3) of the ionian mode the root frequency, and maintaining the same intervals between the chromatic numbers. Therefore, the phrygian scale contains (1)(♭2)(♭3)(4)(5)(♭6)(♭7)(1), where the first (1) indicates root and the final (1) indicates root of the next octave.

Lydian Scale

The lydian scale may be obtained by making the (4) of the ionian mode the root frequency, and maintaining the same intervals between the chromatic numbers. Therefore, the lydian scale contains (1)(2)(3)(♯4)(5)(6)(7)(1), where the first (1) indicates root and the final (1) indicates root of the next octave.

Mixolidian Scale

The mixolidian scale may be obtained by making the (5) of the ionian mode the root frequency, and maintaining the same intervals between the chromatic numbers. Therefore, the mixolidian scale contains (1)(2)(3)(4)(5)(6)(♭7)(1), where the first (1) indicates root and the final (1) indicates root of the next octave.

Aeolian Scale

The aeolian scale may be obtained by making the (6) of the ionian mode the root frequency, and maintaining the same intervals between the chromatic numbers. Therefore, the aeolian scale contains (1)(2)(♭3)(4)(5)(♭6)(♭7)(1), where the first (1) indicates root and the final (1) indicates root of the next octave.

Locrian Scale

The locrian scale may be obtained by making the (7) of the ionian mode the root frequency, and maintaining the same intervals between the chromatic numbers. Therefore, the locrian scale contains (1)(♭2)(♭3)(4)(♭5)(♭6)(♭7)(1), where the first (1) indicates root and the final (1) indicates root of the next octave.

Summary of the Diatonic Modes

Let us now summarize the 7 diatonic scales in the following table.

scale	difference from ionian scale
lydian	(♯4)
ionian	
mixolidian	(♭7)
dorian	(♭7)(♭3)
aeolian	(♭7)(♭3)(♭6)
phrygian	(♭7)(♭3)(♭6)(♭2)
locrian	(♭7)(♭3)(♭6)(♭2)(♭5)

From an aesthetic point of view, the lydian scale has the brightest sound, with the sound getting darker as we go further down the above table.

Definition 3. *dissonant scale;*
If a scale contains a (♭5)*, then that scale is said to be a dissonant scale.*

Definition 4. *major scale;*
If a scale contains a (3)*, then that scale is said to be a major scale.*

Definition 5. *minor scale;*
If a scale contains a $(\flat 3)$*, then that scale is said to be a minor scale.*

1.2.3 Pentatonic Scales

We can construct two very interesting scales that contain only $(\flat 2)(\flat 3)(\flat 5)(\flat 6)(\flat 7)$. If we make the $(\flat 5)$ the root note, then we have the major pentatonic scale. From our understanding of chromatics, the major pentatonic scale contains $(1)(2)(3)(5)(6)(1)$. If we make the $(\flat 3)$ the root note, then we have the minor pentatonic scale. From our understanding of chromatics, the minor pentatonic scale contains $(1)(\flat 3)(4)(5)(\flat 7)(1)$.

1.3 A440 Pitch Standard

Suppose we were to assign letter designations, called notes, to the all of the chromatics from root to root of next octave, as shown in the following table.

chromatic	letter designation
(1)	C
$(\flat 2)$	$C\sharp$ or $D\flat$
(2)	D
$(\flat 3)$	$D\sharp$ or $E\flat$
(3)	E
(4)	F
$(\flat 5)$	$F\sharp$ or $G\flat$
(5)	G
$(\flat 6)$	$G\sharp$ or $A\flat$
(6)	A
$(\flat 7)$	$A\sharp$ or $B\flat$
(7)	B
(1)	C

If, in the above table, we assign a frequency of 440 Hz to the (6) or A, then the (1) or C will have a frequency of approximately 261.626 Hz[2]. This scheme of setting the frequencies of all the chromatics is called the A440 pitch standard.

It would be helpful to note that $E\sharp = F$, $F\flat = E$, $B\sharp = C$, and $C\flat = B$.

1.4 Circles of 4 and 5

1.4.1 Circle of 5

Let us consider a (1), quite clearly the (5) relative to (1) is (5). Since $7+7=14$ and $14 \mod 12 = 2$, it must be that the (5) relative to (5) is (2). Continuing in

[2]We call this note middle C.

this fashion, we may deduce that the chromatics in steps of (5), are as follows;

$$(1)(5)(2)(6)(3)(7)(\flat 5)(\flat 2)(\flat 6)(\flat 3)(\flat 7)(4)(1)$$

This is called the circle of 5.

1.4.2 Circle of 4

Let us consider a (1), quite clearly the (4) relative to (1) is (4). Since $5+5=10$, it must be that the (4) relative to (4) is ($\flat 7$). Continuing in this fashion, we may deduce that the chromatics in steps of (4), are as follows;

$$(1)(4)(\flat 7)(\flat 3)(\flat 6)(\flat 2)(\flat 5)(7)(3)(6)(2)(5)(1)$$

This is called the circle of 4. Notice that the circle of 5 and the circle of 4 are inverses of each other.

1.4.3 Pythagorean Tuning

Now, suppose we were to now define (5) such that the frequency of (5) is $\frac{3}{2}$ times that of (1), and the frequency of (4) is $\frac{4}{3}$ times that of (1). If we were to use this definition of (5) and (4), then we may determine that, according to this scheme and the circle of 5,

chromatic	ratio with respect to root
(1)	1
(5)	$\frac{3}{2}$
(2)	$\frac{9}{8}$
(6)	$\frac{27}{16}$
(3)	$\frac{81}{64}$
(7)	$\frac{243}{128}$
($\sharp 4$)	$\frac{729}{512}$

Similarly, according to this scheme and the circle of 4,

chromatic	ratio with respect to root
(1)	1
(4)	$\frac{4}{3}$
($\flat 7$)	$\frac{16}{9}$
($\flat 3$)	$\frac{32}{27}$
($\flat 6$)	$\frac{128}{81}$
($\flat 2$)	$\frac{256}{243}$
($\flat 5$)	$\frac{1024}{729}$

If (1) is chosen according to the A440 pitch standard, and all of the other chromatics determined according to the above scheme, then it is said that Pythagorean tuning is being used. Aesthetically, it has been found that this tuning scheme defines more harmonious frequencies.

1.5 Keys

Definition 6. *key;*

Given an ionian scale, the note indicated by (1) *is said to be the key of the scale. The key of other diatonic scales is described by the ionian scale from which it was derived.*

1.5.1 Key of C

Suppose we were to construct an ionian scale rooted on the note C. From Sections 1.2.1, 1.2.2, and 1.3, as well as Definition 6 such a scale would be as follows.

$$C \quad D \quad E \quad F \quad G \quad A \quad B \quad C$$

Thus the scales C ionian, D dorian, E phrygian F lydian, G mixolidian, A aeolian, and B locrian are, collectively, 7 different aspects of the key of C.

1.5.2 Keys in Circle of 5

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(1)
C	D	E	F	G	A	B	C
G	A	B	C	D	E	$F\sharp$	G
D	E	$F\sharp$	G	A	B	$C\sharp$	D
A	B	$C\sharp$	D	E	$F\sharp$	$G\sharp$	A
E	$F\sharp$	$G\sharp$	A	B	$C\sharp$	$D\sharp$	E
B	$C\sharp$	$D\sharp$	E	$F\sharp$	$G\sharp$	$A\sharp$	B
$F\sharp$	$G\sharp$	$A\sharp$	B	$C\sharp$	$D\sharp$	$E\sharp$	$F\sharp$
$C\sharp$	$D\sharp$	$E\sharp$	$F\sharp$	$G\sharp$	$A\sharp$	$B\sharp$	$C\sharp$

1.5.3 Keys in Circle of 4

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(1)
C	D	E	F	G	A	B	C
F	G	A	$B\flat$	C	D	E	F
$B\flat$	C	D	$E\flat$	F	G	A	$B\flat$
$E\flat$	F	G	$A\flat$	$B\flat$	C	D	$E\flat$
$A\flat$	$B\flat$	C	$D\flat$	$E\flat$	F	G	$A\flat$
$D\flat$	$E\flat$	F	$G\flat$	$A\flat$	$B\flat$	C	$D\flat$
$G\flat$	$A\flat$	$B\flat$	$C\flat$	$D\flat$	$E\flat$	F	$G\flat$
$C\flat$	$D\flat$	$E\flat$	$F\flat$	$G\flat$	$A\flat$	$B\flat$	$C\flat$